Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-09-22

Conceptual questions

Question 1. Suppose $f(x, y)$ is a function of two variables with $\nabla f(2, 5) = \langle 4, 0 \rangle$. How many unit vectors $\mathbf{u}$ are there such that $D_{\mathbf{u}} f(2, 5)$ is -4? What about -3? 0? 5?

Question 2. Suppose $\nabla f(a, b) = 0$. In the second derivative test as written on page 961 in the textbook, they say "if $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum."

Is it significant that we look at f_{xx} rather than f_{yy}?

Computations

Problem 1. Consider a point $P = (a, b, c)$ on the surface S defined by $xyz = 6$. The tangent plane H at this point meets the coordinate axes at the points $(A, 0, 0), (0, B, 0), (0, 0, C)$. In other words, A, B, C are the x, y, z-intercepts of H, respectively.

(a) Compute each of A, B, C in terms of a, b, c only.

(b) Show that their product ABC is independent of a, b, c. What is it equal to?